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Coherent resonant transmission in temporally periodically driven potential wells: the Fano mirror

Chang Sub Kim and Arkady M Satanin
Department of Physics, Chonnam National University, Kwangju 500-757, South Korea

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Abstract. The electron transmission over an oscillating quantum well is studied perturbatively and numerically in the phase-coherent regime. The dynamically induced multichannel situation gives rise to a pronounced asymmetric resonance and antiresonance structure. The Fano line obtained is attributed to the interaction of the raised virtual discrete energy level with the continuum. Interestingly, the unitarity condition imposes the restriction that the transmission coefficients vanish altogether at the zero energies, leading to the concept of an electronic mirror.

1. Introduction

Recently, a great deal of attention has been paid to the electron transport through dynamically driven quantum structures [1–13]. Photon-assisted tunnelling problems were of particular interest, as they are associated with the possibility of designing future fast electro-optic devices [3, 8, 9]. Physically, important novel effects were suggested such as negative differential resistance and dynamic localization of electrons in the open/closed systems [5, 12].

The quantum structures studied are typically driven by a periodic time-dependent field, $\sim\cos\omega t$, with the spatial restriction that the excited region be smaller than the phase-coherent length of the electron wave functions. In this case, many channels are opened for the electron transmission. The Floquet states are conveniently used to describe the corresponding electron states and it is transparent in this picture that the electron can transfer from an incident central channel E to the side channels, $E \pm n\hbar\omega$ where $n = \pm 1, \pm 2, \ldots$, with finite probabilities. This constitutes a fundamental difference from the static case where only one channel is allowed for the electron transmission.

In this work we investigate the resonance structure of electron transmission through a harmonically driven quantum well. The objective is to provide further insight into the multichannel transmission problems of current interest, induced by dynamic driving fields, using the theoretical model considered. To simplify our analysis we work in the short-range interaction limit, i.e. assuming $a \ll \lambda$ where a is the well width and λ is the characteristic electron wavelength. With this model one can still understand the basic aspects of the nonstationary tunnelling.

This paper is organized as follows. In section 2 we derive a formal expression for the transmission amplitude under the multichannel conditions. Then, a theoretical scheme for use of the finite-channel approximation is provided in section 3. In section 4 the results obtained from both the perturbation method and the numerical calculation are given with detailed analyses. The conclusions are drawn in section 5. A finite-channel approximation procedure is developed in appendix A. In appendix B the unitarity conditions are obtained.

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2. Multichannel transmission

We consider an electron characterized by an effective mass m^* exposed to a static potential $U(x)$, that may represent the mean-field potential for quantum wells, and a time-dependent potential $V(x)\cos\omega t$. The corresponding Hamiltonian is written as

$$H = \frac{p^2}{2m^*} + U(x) + V(x)\cos\omega t \tag{1}$$

which satisfies

$$H(t + 2\pi/\omega) = H(t). \tag{2}$$

The periodic property, equation (2), allows use of the Floquet state for the electron wave function in the form [14]

$$\Psi_E(x,t) = \sum_{n=-\infty}^{\infty} \mathrm{e}^{-(\mathrm{i}/\hbar)(E+n\hbar\omega)t}\psi_n(x) \tag{3}$$

where E is the quasienergy. Equation (3) imposes the restriction that $\Psi_{E+n\hbar\omega} = \Psi_E$; accordingly one has to only consider the electron energy in the first energy window $(0, \hbar\omega)$ or first Brillouin zone. Other energy windows, $(n\hbar\omega, (n+1)\hbar\omega)$ with $n = \pm 1, \pm 2, \ldots$, may be treated in the extended zone scheme. A direct substitution of equation (3) into the nonstationary Schrödinger equation brings out the equivalent equation for the amplitudes $\psi_n(x)$ to obey:

$$\left(\frac{p^2}{2m^*} + U(x)\right)\psi_n + \frac{V(x)}{2}(\psi_{n+1} + \psi_{n-1}) = (E + n\hbar\omega)\psi_n. \tag{4}$$

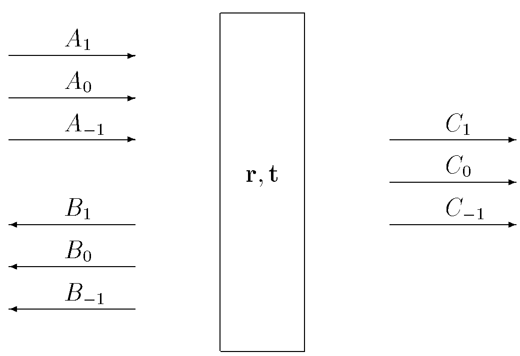

Figure 1. A schematic diagram of the multichannel transmission induced by the harmonically time-dependent fields, where only three channels are drawn as an illustration; **r** and **t** denote the reflection matrix and transmission matrix, respectively.

As a simple model, we take a square well with a depth U_0 and a width in the range $-a/2 \leqslant x \leqslant a/2$, and the time-dependent field $V_0\cos\omega t$, where V_0 is constant, is assumed to be present inside the well. We further argue that the parameters of the quantum well are chosen such that only a single shallow level exists, for calculational convenience, which allows us to use the approximation $U(x) \approx -U_0 a\delta(x)$. So, the interactions of an electron with the quantum well in equation (4) may be represented as

$$U(x) = -\frac{\hbar^2}{m^*}u\delta(x) \qquad V(x) = -2\frac{\hbar^2}{m^*}v\delta(x) \tag{5}$$

where $u \equiv U_0 m^* a/\hbar^2$ and $v \equiv V_0 m^* a/2\hbar^2$. Then, the harmonic functions are specified in each region as

$$\psi_n = \begin{cases} A_n e^{ik_n x} + B_n e^{-ik_n x} & x < 0 \\ C_n e^{ik_n x} & x > 0 \end{cases} \tag{6}$$

where

$$k_n = \hbar^{-1}\sqrt{2m^*(E + n\hbar\omega)}$$

is the electron wave vector. This situation is depicted in figure 1 schematically. The wave vector of the electron with quasienergy E is real for $E + n\hbar\omega > 0$ and is imaginary for quasienergy giving $E + \hbar\omega n < 0$; in this case $k_n = i|k_n|$. We shall call the solutions with real k_n the *open channels* and the ones with imaginary k_n the *closed channels* (evanescent modes).

One can obtain the following matching conditions at $x = 0$ from equation (4):

$$\psi_n(0^+) - \psi_n(0^-) = 0 \qquad \psi_n'(0^+) - \psi_n'(0^-) = -2u\psi_n - 2v(\psi_{n+1} + \psi_{n-1}). \tag{7}$$

The above equation (7) gives the connections among the amplitude vectors $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ as

$$\boldsymbol{B} = \mathbf{r}\boldsymbol{A} \qquad \text{and} \qquad \boldsymbol{C} = \mathbf{t}\boldsymbol{A} \tag{8}$$

where the transmission ($\mathbf{t}$) and reflection ($\mathbf{r}$) matrices are defined to be

$$\mathbf{t} = i\underline{\ell}^{-1}\mathbf{k} \qquad \text{and} \qquad \mathbf{r} = \mathbf{t} - \mathbf{1}. \tag{9}$$

In the above, $\mathbf{1}$ is the identity matrix and the matrices $\underline{\ell}$ and $\mathbf{k}$ are given as

$$\ell_{n,n'} = (ik_n + u)\delta_{n,n'} + v(\delta_{n,n'+1} + \delta_{n,n'-1}) \tag{10}$$

and

$$\mathbf{k}_{n,n'} = k_n \delta_{n,n'}. \tag{11}$$

The above equation (9) constitutes the formal solution of the single-well scattering problem.

To demonstrate its utility, we apply our theory to the static case. For the stationary potential, i.e. when $V_0 = 0$, we only have to take into account one channel ($n = 0$), and from equation (9) it follows that

$$t = \frac{ik_0}{ik_0 + u} \qquad r = t - 1. \tag{12}$$

The scattering amplitudes possess a pole at energy $E_p = -\hbar^2 u^2/2m^*$ which is the bound-state energy of the electron in the well [15]. Also, one can see from equation (12) that the transmission ($|t|^2$) and reflection ($|r|^2$) coefficients are monotonically increasing and decreasing functions of energy for $E > 0$, respectively, which is the characteristic feature of the short-range interaction potential.

In a general nonstationary situation, one has to invert the matrix $\underline{\ell}$ by employing proper numerical methods. However, using the tridiagonal structure of the matrix $\underline{\ell}$, it is possible to invert it analytically and thus to obtain the amplitudes of reflection and transmission. In particular, helpful information can be obtained when a finite number of channels ψ_n, $n = 0, \pm 1, \pm 2, \ldots, \pm N$, are taken into account. In appendix A we develop this procedure.

Also, one can make a simple connection between our formalism and the Landauer–Büttiker formula. Since the transmission coefficient, $T_{n'n}$ is defined to be the ratio of the incident current flux through channel n over the outgoing flux through channel n', it can be easily obtained that

$$T_{n'n} = \frac{k_{n'}}{k_n}|t_{n'n}|^2. \tag{13}$$

Then, the total transmission coefficient at energy E is given by

$$T(E) = \sum_{n'} T_{n'0}(E) \tag{14}$$

where it should be noticed that only the single summation over all outgoing channels is performed for a fixed incident wave ψ_0.

3. Results and discussion

First, we present the perturbation results from the three-channel approximation. We have also checked them numerically by choosing many channels for the given parameters. The finite-channel approximation used here is equivalent to the weak-coupling limit physically [16]:

$$\frac{\hbar^2 v^2}{2m^*} \ll \hbar\omega.$$

Having in mind that the incoming wave is ψ_0, we write the matrix $\underline{\ell}$ centred around the $n = 0$ column and row as

$$\underline{\ell} = \begin{pmatrix} \mathrm{i}k_1 + u & v & 0 \\ v & \mathrm{i}k_0 + u & v \\ 0 & v & \mathrm{i}k_{-1} + u \end{pmatrix}. \tag{15}$$

By inverting $\underline{\ell}$, we extract the transmission amplitude $t_{00}(E)$ from equation (9), which characterizes the direct tunnelling of an electron with quasienergy E between channels $n = 0$ as

$$t_{00}(E) = \frac{\mathrm{i}k_0}{\mathrm{i}k_0 + u - v(1/[\mathrm{i}k_1 + u])v - v(1/[\mathrm{i}k_{-1} + u])v}. \tag{16}$$

The structure of the denominator of equation (16) suggests that a virtual transition takes place from the channel $n = 0$ to the channels $n = \pm 1$ in this tunnelling process.

Interestingly, when the electron is incident over the oscillating well through a particular quasienergy interval, $(0, \hbar\omega)$, the transmission amplitude $t_{00}(E)$ becomes identically zero at a specific incident energy,

$$E_0 = \hbar\omega - \frac{\hbar^2 u^2}{2m^*} \tag{17}$$

which lies in the continuum for the shallow level considered or in the high-frequency limit. This result is obtained from equation (16) using the fact that k_0 is real (open channel) but $k_{-1} = \mathrm{i}|k_{-1}|$ (closed channel) in the chosen energy window. Also, the pole E_p that makes the denominator of equation (16) vanish is determined to be

$$E_p = \widetilde{E} - \mathrm{i}\Gamma \tag{18}$$

where

$$\tilde{E} \simeq \hbar\omega - \frac{\hbar^2 u^2}{2m^*}\left(1 - \frac{2v^2}{\tilde{k}_0^2 + u^2}\right) \qquad \Gamma \simeq \frac{\hbar^2 \tilde{k}_0 u v^2}{2m^*(\tilde{k}_0^2 + u^2)}$$

with

$$\tilde{k}_0 \equiv \sqrt{\frac{2m^*}{\hbar^2}(\hbar\omega - \hbar^2 u^2/2m^*)}.$$

Accordingly, we have found that the amplitude of equation (16) satisfies

$$t_{00}(E) \sim \frac{E - E_0}{E - \widetilde{E} + \mathrm{i}\Gamma}. \tag{19}$$

The resulting transmission structure manifests the Fano asymmetric resonance structure: the transmission amplitude possesses a zero (*antiresonance*) in real energy E_0 and a nearby pole (*resonance*) in complex energy E_p, where $0 < E_0, \tilde{E} < \hbar\omega$. This is due to the interaction of the virtual discrete level, raised up by $\hbar\omega$ from the bound state of the well considered, with the continuum. This unusual resonance structure of the paired resonance and antiresonance was studied by Fano in his pioneering work on the autoionization problem in atomic excitation spectra [17]. In the static low-dimensional quantum structures, the Fano resonances have been discussed at length [18, 19]. If the zero energy, E_0, is real (it can be complex in a different energy window), the transmission amplitude equals zero at $E = E_0$, and the amplitude shows a peak at $E = \tilde{E}$ with a width Γ near and to the right of E_0. We can also obtain the transmission amplitude $t_{10}(E)$ from the channel $n = 0$ to the open channel $n = 1$ in the same energy window $(0, \hbar\omega)$.

In figure 2 the resulting Fano transmission lines are illustrated, where one can see clearly the paired antiresonance and resonance structures. In figure 2(a) the antiresonance occurs at $E = E_0 \simeq 0.796\,\hbar\omega$. And, the resonance occurs at $E = \tilde{E} \simeq 0.817\,\hbar\omega$, having a peak value of 0.985, with the width $\Gamma \simeq 0.009\,\hbar\omega$. The dotted curve depicts the monotonically increasing transmission with energy in the static case. A similar feature is seen in figure 2(b) although the overall scale is much smaller than the one in figure 2(a). It is suggestive that $T_{10}(E)$ has the same zero energy as $T_{00}(E)$, which reflects the fact that our construction satisfies the conservation of electron current, which reads

$$T_{00} + T_{10} = \mathrm{Re}(t_{00}). \tag{20}$$

The proof of equation (20) is given in appendix B. Equation (20) states that if $T_{00}(E) = 0$ at $E = E_0$, then $T_{10}(E)$ also vanishes at the same energy: the unitarity condition closes the transition from $n = 0$ to $n = 1$. Therefore, when an electron is incident upon the nonstationary quantum well at the particular energy E_0, the well plays the role of a perfect mirror, i.e. the Fano *mirror*. The total transmission $T(E)$ is not drawn, since there is no appreciable difference from $T_{00}(E)$. Also, we have confirmed this analytic result numerically by taking into account a large number of channels in the parameter ranges where the finite-channel model works.

Next, we shall carry out the general analysis of a nonstationary single-well system. Using the tridiagonal structure of the matrix $\underline{\ell}$, one can invert it to obtain the general structure of the transmission amplitudes $\mathbf{t}$. After inverting $\underline{\ell}$, we managed to obtain a formal representation of the component $t_{00}(E)$ from equation (9) as follows:

$$t_{00}(E) = \frac{\mathrm{i}k_0\Delta_1\Delta_{-1}}{(\mathrm{i}k_0 + u)\Delta_1\Delta_{-1} - v^2(\Delta_2\Delta_{-1} + \Delta_{-2}\Delta_1)} \tag{21}$$

where $\Delta_{\pm s}$ are the determinants of the sub-matrices of $\underline{\ell}$, which are tridiagonal matrices with dimension $(\pm s, \pm\infty) \times (\pm s, \pm\infty)$, respectively, where $s = 0, 1, \ldots$. Here, it is important to notice that the choice of the reference column (or row) $n = 0$ in the expansion of $\det \underline{\ell}$ is equivalent to specifying an initial harmonic in equation (3) as $\psi(x, 0) \equiv \psi_0(x)$.

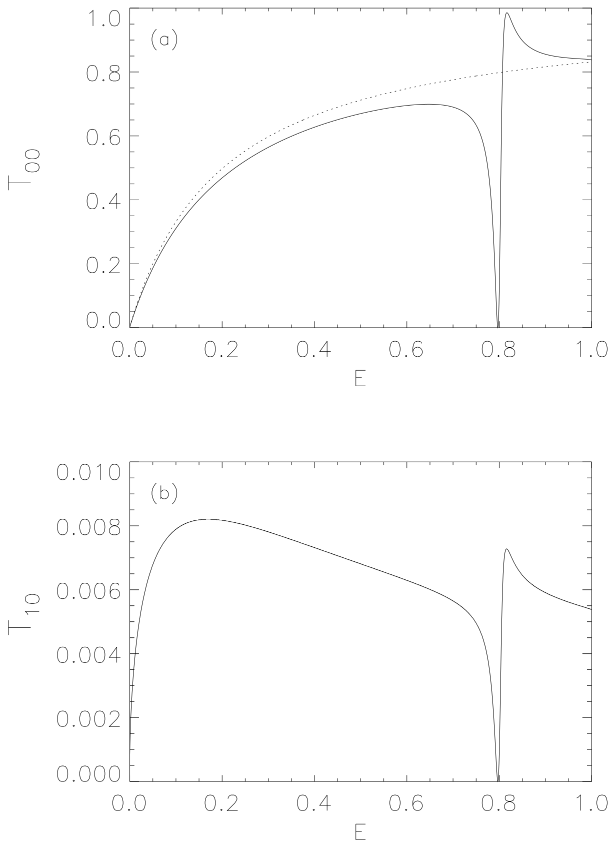

Figure 2. Coefficients of transmission, $T_{n'0}(E)$, equation (13), through the quasienergy Brillouin zone $(0, \hbar\omega)$ where E is in units of $\hbar\omega$: (a) $T_{00}(E)$ where the dotted curve shows the static case; (b) $T_{10}(E)$, where the parameters were chosen as $u = 0.45$ and $v = 0.15$, both in units of $\sqrt{2m^*\omega/\hbar}$. The total transmission $T(E)$ is not drawn since there is no appreciable difference from $T_{00}(E)$.

We now find the zero energy of the transmission amplitude $t_{00}(E)$ in the quasienergy interval $(0, \hbar\omega)$. In this energy window, wave vectors k_n are real for $n > 0$ and imaginary for $n < 0$; consequently it turns out that the determinants Δ_{+s} are complex and Δ_{-s} are real for $s \geqslant 1$. Thus, the vanishing condition of Δ_{-1}, i.e.

$$\Delta_{-1} = 0 \tag{22}$$

gives rise to the zero energy of $t_{00}(E)$ for real E. In order to find the zero of Δ_{-1} we further rewrite equation (22) by using the tridiagonal structure of the matrix as follows:

$$(-|k_1| + u)\Delta_{-2} - v^2\Delta_{-3} = 0 \tag{23}$$

which gives

$$E_0 = \hbar\omega - \frac{\hbar^2}{2m^*}\left(u + v^2\frac{\Delta_{-3}}{\Delta_{-2}}\right)^2. \tag{24}$$

Then, we can perform the perturbation expansion for the right-hand side of equation (24) in the weak-coupling limit. For the three-channel approximation we have recovered the result given in equation (17). And, for five-channel approximation we obtain the shift of the zero energy along the real quasienergy axis as follows:

$$E_0 = \hbar\omega - \frac{\hbar^2}{2m^*}\left(u + \frac{v^2}{|\tilde{k}_{-2}| - u}\right)^2 \tag{25}$$

where

$$|\tilde{k}_{-2}| \equiv \sqrt{\frac{2m^*}{\hbar^2}\left(\hbar\omega + \frac{\hbar^2 u^2}{2m^*}\right)}.$$

Considering equation (22) for the determination of zeros is equivalent to solving the Schrödinger equation (4). Here, we provide the wave function corresponding to equation (22) at $E = E_0$. We have obtained that $\psi_n(x) = 0$ at all x for $n > 0$, and

$$\psi_0(x) = \begin{cases} A_0 \sin(k_0 x) & x < 0 \\ 0 & x > 0 \end{cases} \tag{26}$$

where $k_0 = \sqrt{2m^* E_0/\hbar^2}$, and

$$\psi_{-n}(x) = \begin{cases} B_{-n} e^{|k_{-n}|x} & x < 0 \\ C_{-n} e^{-|k_{-n}|x} & x > 0 \end{cases} \tag{27}$$

where $|k_{-n}| = \sqrt{2m^*(n\hbar\omega - E_0)/\hbar^2}$. It is seen that when the total reflection occurs the incident wave with wave vector k_0 forms a standing wave ψ_0 with the node at the interface $x = 0$ for $x < 0$. On the other hand, all other nonvanishing channels belong to the evanescent modes.

One can also find the expressions for complex Fano poles from equation (21), and a shift has been obtained in the complex energy plane with the order-of-v^4 correction to equation (18). Physically the poles of the scattering matrix (21) specify the quasibound states with finite lifetimes $\sim\hbar/\Gamma$.

Although we have considered so far a specific incident electron energy interval, $(0, \hbar\omega)$, the general many-channel amplitude equation (21) possesses a zero energy in each interval $(n\hbar\omega, (n+1)\hbar\omega)$ in the extended band scheme. We found that the zero energy is real only for the quasienergy interval $(0, \hbar\omega)$. For example, if we consider the energy window $(\hbar\omega, 2\hbar\omega)$, we obtain the complex E_0 from equation (22):

$$E_0 = 2\hbar\omega - \frac{\hbar^2}{2m^*}\left(u + v^2\left(\frac{u - \mathrm{i}k_{-1}}{u^2 + k_{-1}{}^2} + \frac{\Delta_{-4}}{\Delta_{-3}}\right)\right)^2. \tag{28}$$

In order to understand the meaning of the complex energy more clearly, we expand E_0 up to order v^2 as follows:

$$E_0 \simeq 2\hbar\omega - \frac{\hbar^2}{2m^*}\left(u^2 - 2uv^2\left(\frac{1}{\mathrm{i}k_{-1} + u} + \frac{1}{-|k_{-2}| + u}\right)\right). \tag{29}$$

This shows that the zero of $t_{00}(E)$ is displaced from the real axis to the complex quasienergy plane. This situation can be understood physically as follows. We have obtained that the incident wave with k_0 can be scattered into another open channel of k_{-1} with the same energy in the energy window considered. So, in this case two waves with different wave vectors k_0 and k_{-1} coexist in the reflected region. Therefore, it is impossible for these waves to interface simultaneously at the boundaries because k_0 and k_{-1} are incommensurable. This means that the reflection is imperfect for the chosen quasienergy window.

Finally, it seems worthwhile to mention the possible mechanisms for the time-dependent driving fields. In reference [20], Jonson discussed the nondispersive localized phonons, the electromagnetic field of a laser beam, and the oscillating ac voltage as possible sources, depending on the physical systems under consideration. We found that the coupling constant derived in reference [20] corresponds to our $(V_0/\hbar\omega)^2$ roughly (see [16]). Thus, one may find an estimate for the amplitude of the driving field using the coupling constants studied in [20]. However, it should be noted that there is an essential difference between our work and the model given in [20] where a double-barrier structure was considered. The resonance in such structures is Breit–Wigner-type, and the transfer of resonant tunnelling from the central peak to the satellite peaks was predicted due to the opening up of the sidebands. In our case, we studied the attractive potential well with a bound state; consequently the Fano resonances do appear due to the interaction of the virtual discrete level with the continuum, providing a mechanism for an *electronic mirror*.

4. Conclusion

We have studied the one-dimensional electron transmission through an oscillating semiconductor structure in detail. Due to the presence of the harmonic driving fields, the incident electron finds many channels via which it can pass though the system. We have obtained an analytic expression for the transmission coefficient matrix and have analysed it perturbatively. Also, we have carried out a numerical evaluation by incorporating a large number of channels to find a good agreement with the analytic finite-channel approximation. Our results show the interesting Fano resonance structure for the chosen parameters, i.e. an asymmetrically paired resonance and antiresonance line-shape. In particular, we have developed the concept of the Fano mirror by noticing the quenching of the electron transmission at the zero energies. When the electron is incident on the system with quasienergies above the energy of one quantum associated with the driving fields, the mirror becomes imperfect. As an extension of this work, it would be very interesting to investigate whether it is possible to confine electrons dynamically between two Fano mirrors. The results will be reported elsewhere [21].

A conventional quantum well is three dimensional; the transverse motion participates in addition to the vertical transport that we have considered in the present work. However, it is well known that the total transport can be reduced to a one-dimensional problem as long as interactions that change the parallel momentum are suppressed [22]. Thus, the signature of Fano resonances predicted may still be present in quantum wells within the assumed ballistic limit at low temperatures.

Acknowledgments

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Appendix A. The finite-channel approximation

In this section a perturbation method of finding solution to the difference equation (4) is developed; the equation is rewritten in the form

$$H_0\psi_n + V(x)(\psi_{n+1} + \psi_{n-1}) = (E + n\hbar\omega)\psi_n \tag{A1}$$

where $H_0 = p^2/2m^* + U(x)$ is treated as the unperturbed part of Hamiltonian. We use the idea of an operator expansion as an infinite fraction. To this end, we introduce the auxiliary operators R_s^+ and R_s^- defined through the relations

$$\psi_{s+1} = R_s^+ \psi_s \qquad \text{where } \psi_s \equiv \psi_n \qquad \text{for } n > 0 \tag{A2}$$

$$\psi_{-(s+1)} = R_{-s}^+ \psi_{-s} \qquad \text{where } \psi_{-s} \equiv \psi_n \qquad \text{for } n < 0. \tag{A3}$$

For the $n = 0$ case, by substituting equations (A2) and (A3) into equation (A1) we get

$$\left\{H_0 + V(x)(R_0^+ + R_0^-)\right\} \psi_0 = E\psi_0. \tag{A4}$$

Accordingly, if R_0^+ and R_0^- are given, the harmonic ψ_0 can be determined from equation (A4). Next, we consider the case where $n > 0$. By plugging equation (A2) into equation (A1) we can obtain

$$(E + s\hbar\omega - H_0 - VR_s^+)\psi_s = V\psi_{s-1}. \tag{A5}$$

And, carrying out the relabelling $s \to s - 1$ in equation (A2) gives

$$\psi_s = R_{s-1}^+ \psi_{s-1}. \tag{A6}$$

After substituting equation (A6) to equation (A5) and rearranging, one can identify that

$$R_{s-1}^+ = (E + s\hbar\omega - H - VR_s^+)^{-1} V. \tag{A7}$$

Similarly, for $n < 0$ the equation for R_{-s}^- is obtained as

$$R_{-s+1}^- = (E - s\hbar\omega - H - VR_{-s}^-)^{-1} V. \tag{A8}$$

Now, let us truncate the infinite vector $(\ldots, \psi_{-n}, \ldots, \psi_{-1}, \psi_0, \psi_1, \ldots, \psi_n, \ldots)$ to a finite-dimensional vector, $(\psi_{-N}, \ldots, \psi_{-1}, \psi_0, \psi_1, \ldots, \psi_N)$, assuming that all ψ_n with $|n| > N$ vanish identically. This truncation corresponds to the $(2N + 1)$-channel approximation. Then, equations (A2) and (A3) ensure that

$$R_N^+ = 0 \qquad \text{and} \qquad R_{-N}^- = 0. \tag{A9}$$

Starting from equation (A9) for a chosen N, one can generate all R_s^+ and R_{-s}^- with $s = N-1, N-2, \ldots, 0$. For instance, R_{N-1}^+ is obtained from equation (A7) by substituting in $R_N^+ = 0$:

$$R_{N-1}^+ = (E + N\hbar\omega - H_0)^{-1} V. \tag{A10}$$

By substituting R_{N-1}^+ into equation (A7), again for $s = N - 1$, one can obtain R_{N-2}^+. By carrying out this recursive substitution one can produce all $R_{N-1}^+, \ldots, R_1^+, R_0^+$. Similarly, starting from $R_{-N}^- = 0$ one can obtain the operators $R_{-N+1}^-, \ldots, R_{-1}^-, R_0^-$.

Next, with use of the R_s^+ and R_{-s}^- obtained, the harmonics ψ_n can be determined. Putting R_0^+ and R_0^- in equation (A4) we can find the wave function ψ_0. Once ψ_0 is specified, ψ_s and ψ_{-s} are determined from equations (A2) and (A3) by successively applying R_s^+ and R_{-s}^-, respectively, for $s = 1, 2, \ldots, N$.

As a concrete example, we consider the three-channel approximation ($N = 1$). In this case we set $\psi_{-n} \equiv 0 \equiv \psi_n$ for $n > 1$ and $R_1^+ = 0 = R_{-1}^-$; accordingly,

$$R_0^+ = (E + \hbar\omega - H_0)^{-1} V \tag{A11}$$

and

$$R_0^- = (E - \hbar\omega - H_0)^{-1} V. \tag{A12}$$

Then, the wave function ψ_0 is obtained from equation (A4) as

$$\left\{H_0 + V(x)\frac{1}{E + \hbar\omega - H_0}V(x) + V(x)\frac{1}{E - \hbar\omega - H_0}V(x)\right\}\psi_0 = E\psi_0. \tag{A13}$$

Also, from equations (A2) and (A3) we find

$$\psi_1 = \left\{ \frac{1}{E + \hbar\omega - H_0} \right\} V(x)\psi_0 \tag{A14}$$

$$\psi_{-1} = \left\{ \frac{1}{E - \hbar\omega - H_0} \right\} V(x)\psi_0. \tag{A15}$$

Similarly, for the five-channel approximation ($N = 2$), we find the equation for ψ_0:

$$\begin{aligned}\Bigg\{ H_0 &+ V(x)\frac{1}{E + \hbar\omega - H_0}V(x) \\ &+ V(x)\frac{1}{E + \hbar\omega - H_0}V(x)\frac{1}{E + 2\hbar\omega - H_0}V(x)\frac{1}{E + \hbar\omega - H_0}V(x) \\ &+ V(x)\frac{1}{E - \hbar\omega - H_0}V(x) \\ &+ V(x)\frac{1}{E - \hbar\omega - H_o}V(x)\frac{1}{E - 2\hbar\omega - H_0}V(x)\frac{1}{E - \hbar\omega - H_0}V(x)\Bigg\}\psi_0 \\ &= E\psi_0. \end{aligned} \tag{A16}$$

And, with use of ψ_0 the other components of the harmonics are determined from

$$\psi_{\pm 1} = \left\{ \frac{1}{E \pm \hbar\omega - H_0} + \frac{1}{E \pm \hbar\omega - H_0}V(x)\frac{1}{E \pm 2\hbar\omega - H_0} \right\} V(x)\psi_0 \tag{A17}$$

$$\psi_{\pm 2} = \left\{ \frac{1}{E \pm \hbar\omega - H_0} \right\} V(x)\psi_0. \tag{A18}$$

The expansion developed indicates that the truncation of the infinite wave vector $\{\psi\}$ at the finite-number of components, the finite-channel approximation, is valid when $V/\hbar\omega$ is sufficiently small that the terms apart from H_0 can be treated as a perturbation. For example, in the three-channel approximation, equation (A13), the perturbed Hamiltonian, is given by

$$H' = V(x)\left(\frac{1}{E + \hbar\omega - H_0} + \frac{1}{E - \hbar\omega - H_0} \right) V(x) \tag{A19}$$

where one can see that the characteristic energy scale of the denominator is $\hbar\omega$ for electron quasienergy $E \sim \hbar\omega$ and the shallow level considered in the text. For the five-channel approximation case, the perturbed terms are seen to be of the order of $(V/\hbar\omega)^2$ in equation (A16).

Appendix B. The unitarity condition

We provide in this appendix the unitarity conditions that the scattering matrices satisfy. The current carried by each harmonic ψ_n is given by

$$j_n = -e\frac{\hbar}{m^*}\,\mathrm{Im}\left(\psi_n^* \frac{\partial}{\partial x}\psi_n \right). \tag{B1}$$

For an open channel, $\psi_n = A_n \mathrm{e}^{\mathrm{i}k_n x}$, this becomes

$$j_n = -e\frac{\hbar}{m^*}|A_n|^2 k_n \tag{B2}$$

and for a closed channel it vanishes identically. Then, the total current incident through many open channels is written as

$$I_i = -e\frac{\hbar}{m^*}\sum_{n=0}^{\infty}|A_n|^2 k_n. \tag{B3}$$

In general, the conservation of electron current through the system requires that $I_i = I_r + I_t$ where I_r and I_t denote the reflected and transmitted current, respectively:

$$\sum_n |A_n|^2 k_n = \sum_n |B_n|^2 k_n + \sum_n |C_n|^2 k_n. \tag{B4}$$

Equation (B4) can be rewritten in matrix form as

$$\boldsymbol{A}^\dagger \mathbf{k} \boldsymbol{A} = \boldsymbol{B}^\dagger \mathbf{k} \boldsymbol{B} + \boldsymbol{C}^\dagger \mathbf{k} \boldsymbol{C} \tag{B5}$$

which, using equations (8) and (9), can be further converted into

$$\mathbf{t}^\dagger \mathbf{k} \mathbf{t} + \mathbf{r}^\dagger \mathbf{k} \mathbf{r} = \mathbf{k} \tag{B6}$$

implying that the scattering matrix is unitary. For the short-range potential considered we have from equation (9)

$$\mathbf{r} = \mathbf{t} - \mathbf{1}. \tag{B7}$$

Accordingly, equation (B6) can be cast into the form

$$\mathbf{t}^\dagger \mathbf{k} \mathbf{t} = \frac{1}{2}(\mathbf{k}\mathbf{t} + \mathbf{k}^\dagger \mathbf{t}). \tag{B8}$$

The above equation (B8) can be spelled out in detail in the three-channel approximation as follows:

$$k_0|t_{00}|^2 + k_1|t_{10}|^2 = k_0\,\mathrm{Re}(t_{00}) \tag{B9}$$

$$k_1 t_{10}^* t_{11} + k_0 t_{00}^* t_{01} = \frac{1}{2}(k_1 t_{01} + k_0 t_{10}^*) \tag{B10}$$

$$k_0|t_{01}|^2 + k_1|t_{11}|^2 = k_1\,\mathrm{Re}(t_{11}) \tag{B11}$$

$$k_1 t_{11}^* t_{10} + k_0 t_{00} t_{01}^* = \frac{1}{2}(k_1 t_{10} + k_0 t_{01}^*). \tag{B12}$$

In particular, equation (B9) is the one considered in the text explicitly, after dividing it by k_0 and with the use of equation (13), to show the structure of the unitarity when the harmonic $n = 0$ is chosen as the incident channel.

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[16] In appendix A, we show that a truncation of the infinite system of equations for the amplitudes $\{\psi_n\}$ at a finite number is valid for small $V_0/\hbar\omega$. It can be shown that the coupling constant $g \equiv (\hbar^2 v^2/2m^*)/\hbar\omega = (\pi^2/16)(\hbar\omega/E_b)(V_0/\hbar\omega)^2$ where $E_b = (\hbar^2/2m^*)(\pi/a)^2$; accordingly, the finite-channel approximation is related to the weak-coupling limit, $\hbar^2 v^2/2m^* \ll \hbar\omega$.

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[22] See, for instance,

Tsu R and Esaki L 1973 *J. Appl. Phys.* **22** 562

Also, see references [1, 3] in this paper for the time-dependent case.